Endogenous Mobility

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AKM in the Presence of Endogenous Mobility

$$\ln w_{it} \equiv y_{it} = X_{it}\beta + \theta_i + \psi_{\mathrm{J}(i,t)} + \varepsilon_{it}$$

- **Goal:** Extend applications of the Abowd-Kramarz-Margolis (Abowd et al. 1999) decomposition
- **Problem:** Structural interpretations rely on the assumption that job mobility is exogenous to ε
- **The Approach:** Model the *realized mobility network* to correct for endogeneity bias

Role of Network Methods

1. **Modeling:** Model selection of employment relationships as an evolving bipartite graph.

2. **Computation:** Exploit network structure to facilitate computation (graph coloring).

3. **Interpretation:** Connection between least-squares normal equations and the bipartite adjacency matrix.

Results

- Endogenous mobility induces a bias that *compresses* worker and firm wage heterogeneity.
- Structural match effects negatively correlated with structural worker and firm effects
- Match effects reduce the part of variation previously explained by firm effects.
- Positive assortative matching on wage components is actually due to sorting on match quality.
- AKM and structural wage components are positively correlated

- Separation and assignment depend on worker type, firm type and match quality
- AKM residual has no correlation with structural wage components

Estimating Individual and Employer Wage Effects

- The AKM (1999) specification for the wage determination equation with individual and employer heterogeneity

$$y = X\beta + D\theta + F\psi + \varepsilon$$

- where y is the $[N \times 1]$ stacked vector of log wage outcomes y_{it}, now sorted by t, then i
- X is the $[N \times k]$ design matrix of observable individual and employer time-varying characteristics
- D is the $[N \times I]$ design matrix for the individual effects
- F is the $[N \times J]$ design matrix for the employer effects (non-employment suppressed)
- ϵ is the $[N \times 1]$ vector of statistical errors
- $\begin{bmatrix} \beta^T & \theta^T & \psi^T \end{bmatrix}^T$ are the unknown effects $[k \times 1]$, $[I \times 1]$, and $[J \times 1]$, resp., associated with each of the design matrices.

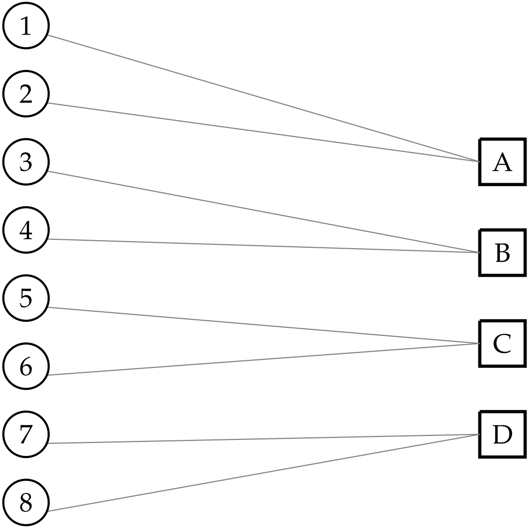
- Solving the fixed-effects moment equations

$$\begin{bmatrix} X^TX & X^TD & X^TF \\ D^TX & D^TD & D^TF \\ F^TX & F^TD & F^TF \end{bmatrix} \begin{bmatrix} \beta \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} X^Ty \\ D^Ty \\ F^Ty \end{bmatrix}$$

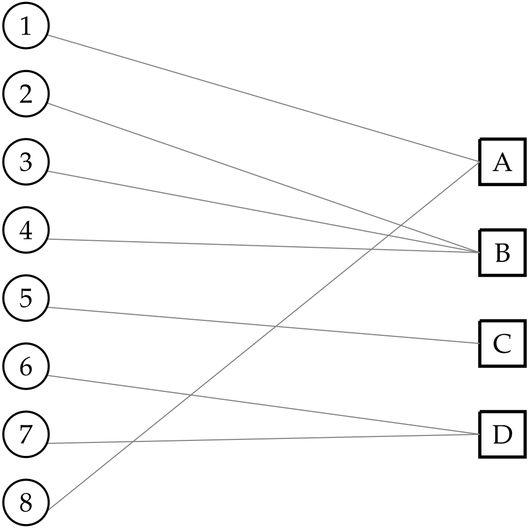
- Identification using graph methods (Abowd et al. 2002)

- Yields estimates of the components of heterogeneity

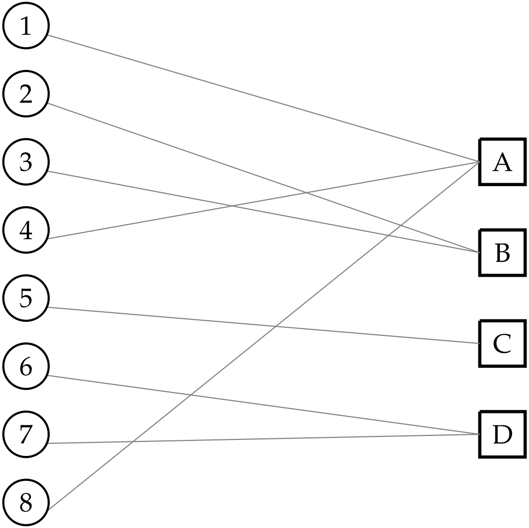
Realized Employment Network, $t = 1$



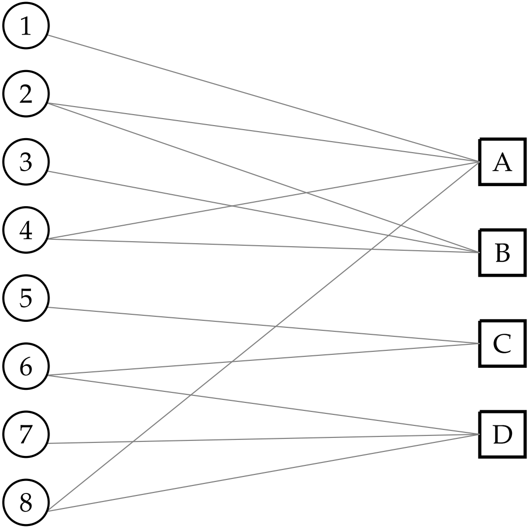
Realized Employment Network, $t = 2$



Realized Employment Network, $t = 3$



Realized Mobility Network



Modeling the Realized Mobility Network

- Populations:
  - Workers: $A = \{1, \ldots, I\}$
  - Employers: $E = \{0, 1, \ldots, J\}$
- Note: A and E are disjoint vertex (node) sets
- $Q = A \times E$ is the set of feasible matches (or edges)
- $M(t)$ is the set of realized employment matches at time t

$$M(t) = \{(i, j) \in Q | j = \mathrm{J}(i, t)\}$$

- Let $B(t)$ be the adjacency matrix representation of $M(t)$

- The "realized employment networks" are snapshots of the labor market at points in time, $B(t_1), ..., B(t_T)$
- The adjacency matrices describe the selection of wage observations for
  - workers
  - firms employers

  from $I \times (J + 1)$ potential outcomes at each moment of time
- We address endogenous selection by jointly modeling wages and the evolution of B

Restating in Terms of the Adjacency Matrix Sequence

- Note that, returning to AKM notation, when the data sort order is t then i,

$$F = \begin{bmatrix} B(1) \\ B(2) \\ \vdots \\ B(T) \end{bmatrix}$$

  where $B(t)$ is the adjacency matrix from the bipartite labor market graph

- We model evolution of $B(t)$ as a Markov process that depends on wage offers.

- Workers, firms, and matches belong to L, M, and Q latent heterogeneity classes
- a_i is a $1 \times L$ indicator of the ability class of worker $i \in \{1, \ldots, I\}$
- b_j is a a $1 \times M$ indicator of the productivity class of employer $j \in \{0, \ldots, J\}$
- k_{ij} is a $1 \times Q$ indicator of the quality of the match between i and j
- Match quality depends on ability and productivity
- Earnings and mobility both depend on all three components

- **Wages**

$$\ln w_{ijt} = \alpha + X_{it}\beta + a_i\theta + b_j\psi + k_{ij}\mu + \varepsilon_{ijt}$$

  where θ, ψ and μ are now vectors of log wage effects
- **Mobility** probability of separation and transition depends on a, b and k

Observed Data, Latent Data and Parameters

- Observed data

$$y_{it} = [\ln w_{it}, X_{it}, s_{it}, m_{it}, i, \mathrm{J}(i,t), \mathrm{J}(i,t+1)]$$

for $i = 1, ..., I$ and $t = 1, ...T$.

- Latent data vector

$$Z = [a_1, \ldots, a_I, b_0, \ldots, b_J, k_{11}, k_{12}, \ldots, k_{1J}, k_{21}, \ldots, k_{IJ}]$$

- Parameter vector

$$\rho^T = \left[\alpha, \beta^T, \theta^T, \psi^T, \mu^T, \sigma, \gamma, \delta, \pi_a, \pi_b, \pi_{k|ab}\right], \rho \in \Theta$$

Complete Data Likelihood Function

$$L\left(\rho|Y,Z\right) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{l} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(\ln w_{i\,\mathrm{J}(i,t)t} - \alpha - X_{it}\beta - a_i\theta - b_{\mathrm{J}(i,t)}\psi - k_{i\,\mathrm{J}(i,t)}\mu\right)^2}{2\sigma^2} \right] \right)^{m_{it}} \\ \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i \rangle \left\langle b_{\mathrm{J}(i,t)} \right\rangle \left\langle k_{i\,\mathrm{J}(i,t)} \right\rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i \rangle \left\langle b_{\mathrm{J}(i,t)} \right\rangle \left\langle k_{i\,\mathrm{J}(i,t)} \right\rangle} \right]^{s_{it}} \\ \times \prod_{t=1}^{T-1} \left[\delta_{\left\langle b_{\mathrm{J}(i,t+1)} \right\rangle | \langle a_i \rangle \left\langle b_{\mathrm{J}(i,t)} \right\rangle \left\langle k_{i\,\mathrm{J}(i,t)} \right\rangle} \right]^{s_{it}} \end{array} \right.$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} \left(\pi_{a\ell}\right)^{a_{i\ell}} \left(\pi_{bm}\right)^{b_{jm}} \left(\pi_{q|\ell m}\right)^{k_{ijq}} \right]$$

Complete Data Likelihood Function

$$L\left(\rho|Y,Z\right) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{l} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(\ln w_{i\,\mathrm{J}(i,t)t} - \alpha - X_{it}\beta - a_i\theta - b_{\mathrm{J}(i,t)}\psi - k_{i\,\mathrm{J}(i,t)}\mu\right)^2}{2\sigma^2} \right] \right)^{m_{it}} \\ \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i\rangle \langle b_{\mathrm{J}(i,t)}\rangle \langle k_{i\,\mathrm{J}(i,t)}\rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i\rangle \langle b_{\mathrm{J}(i,t)}\rangle \langle k_{i\,\mathrm{J}(i,t)}\rangle} \right]^{s_{it}} \\ \times \prod_{t=1}^{T-1} \left[\delta_{\langle b_{\mathrm{J}(i,t+1)}\rangle | \langle a_i\rangle \langle b_{\mathrm{J}(i,t)}\rangle \langle k_{i\,\mathrm{J}(i,t)}\rangle} \right]^{s_{it}} \end{array} \right.$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} \left(\pi_{a\ell}\right)^{a_{i\ell}} \left(\pi_{bm}\right)^{b_{jm}} \left(\pi_{q|\ell m}\right)^{k_{ijq}} \right]$$

Latent Types

Complete Data Likelihood Function

$$L\left(\rho|Y,Z\right) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{l} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(\ln w_{i\,J(i,t)t} - \alpha - X_{it}\beta - a_i\theta - b_{J(i,t)}\psi - k_{i\,J(i,t)}\mu\right)^2}{2\sigma^2} \right] \right)^{m_{it}} \\ \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i \rangle \langle b_{J(i,t)} \rangle \langle k_{i\,J(i,t)} \rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i \rangle \langle b_{J(i,t)} \rangle \langle k_{i\,J(i,t)} \rangle} \right]^{s_{it}} \\ \times \prod_{t=1}^{T-1} \left[\delta_{\langle b_{J(i,t+1)} \rangle | \langle a_i \rangle \langle b_{J(i,t)} \rangle \langle k_{i\,J(i,t)} \rangle} \right]^{s_{it}} \end{array} \right.$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} \left(\pi_{a\ell}\right)^{a_{i\ell}} \left(\pi_{bm}\right)^{b_{jm}} \left(\pi_{q|\ell m}\right)^{k_{ijq}} \right]$$

Latent Types: Workers

Complete Data Likelihood Function

$$L(\rho|Y,Z) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{l} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(\ln w_{i\,\mathrm{J}(i,t)t} - \alpha - X_{it}\beta - a_i\theta - b_{\mathrm{J}(i,t)}\psi - k_{i\,\mathrm{J}(i,t)}\mu\right)^2}{2\sigma^2} \right] \right)^{m_{it}} \\ \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \\ \times \prod_{t=1}^{T-1} \left[\delta_{\langle b_{\mathrm{J}(i,t+1)} \rangle | \langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \end{array} \right.$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} \left(\pi_{a\ell}\right)^{a_{i\ell}} \left(\pi_{bm}\right)^{b_{jm}} \left(\pi_{q|\ell m}\right)^{k_{ijq}} \right]$$

Latent Types: Firms

Complete Data Likelihood Function

$$L(\rho | Y, Z) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{l} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(\ln w_{i\,\mathrm{J}(i,t)t} - \alpha - X_{it}\beta - a_i\theta - b_{\mathrm{J}(i,t)}\psi - k_{i\,\mathrm{J}(i,t)}\mu\right)^2}{2\sigma^2} \right] \right)^{m_{it}} \\ \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \\ \times \prod_{t=1}^{T-1} \left[\delta_{\langle b_{\mathrm{J}(i,t+1)} \rangle | \langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \end{array} \right.$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} \left(\pi_{a\ell}\right)^{a_{i\ell}} \left(\pi_{bm}\right)^{b_{jm}} \left(\pi_{q|\ell m}\right)^{k_{ijq}} \right]$$

Latent Types: Matches

Complete Data Likelihood Function

$$L(\rho|Y,Z) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{l} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(\ln w_{i\,\mathrm{J}(i,t)t} - \alpha - X_{it}\beta - a_i\theta - b_{\mathrm{J}(i,t)}\psi - k_{i\,\mathrm{J}(i,t)}\mu\right)^2}{2\sigma^2} \right] \right)^{m_{it}} \\ \quad \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i \rangle \left\langle b_{\mathrm{J}(i,t)} \right\rangle \left\langle k_{i\,\mathrm{J}(i,t)} \right\rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i \rangle \left\langle b_{\mathrm{J}(i,t)} \right\rangle \left\langle k_{i\,\mathrm{J}(i,t)} \right\rangle} \right]^{s_{it}} \\ \qquad \times \prod_{t=1}^{T-1} \left[\delta_{\left\langle b_{\mathrm{J}(i,t+1)} \right\rangle | \langle a_i \rangle \left\langle b_{\mathrm{J}(i,t)} \right\rangle \left\langle k_{i\,\mathrm{J}(i,t)} \right\rangle} \right]^{s_{it}} \end{array} \right.$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} \left(\pi_{a\ell}\right)^{a_{i\ell}} \left(\pi_{bm}\right)^{b_{jm}} \left(\pi_{q|\ell m}\right)^{k_{ijq}} \right]$$

Mobility

Complete Data Likelihood Function

$$L\left(\rho|Y,Z\right) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{c} \prod_{t=1}^{T}\left(\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{\left(\ln w_{i\,\mathrm{J}(i,t)t}-\alpha-X_{it}\beta-a_i\theta-b_{\mathrm{J}(i,t)}\psi-k_{i\,\mathrm{J}(i,t)}\mu\right)^2}{2\sigma^2}\right]\right)^{m_{it}} \\ \times\prod_{t=1}^{T-1}\left[1-\gamma_{\langle a_i\rangle\left\langle b_{\mathrm{J}(i,t)}\right\rangle\left\langle k_{i\,\mathrm{J}(i,t)}\right\rangle}\right]^{1-s_{it}}\left[\gamma_{\langle a_i\rangle\left\langle b_{\mathrm{J}(i,t)}\right\rangle\left\langle k_{i\,\mathrm{J}(i,t)}\right\rangle}\right]^{s_{it}} \\ \times\prod_{t=1}^{T-1}\left[\delta_{\left\langle b_{\mathrm{J}(i,t+1)}\right\rangle|\langle a_i\rangle\left\langle b_{\mathrm{J}(i,t)}\right\rangle\left\langle k_{i\,\mathrm{J}(i,t)}\right\rangle}\right]^{s_{it}} \end{array} \right.$$

$$\times\prod_{i=1}^{I}\prod_{j=1}^{J}\left[\prod_{\ell=1}^{L}\prod_{m=1}^{M}\prod_{q=1}^{Q}\left(\pi_{a\ell}\right)^{a_{i\ell}}\left(\pi_{bm}\right)^{b_{jm}}\left(\pi_{q|\ell m}\right)^{k_{ijq}}\right]$$

Mobility: Non-separation

Complete Data Likelihood Function

$$L(\rho|Y,Z) \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{l} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(\ln w_{i\,\mathrm{J}(i,t)t} - \alpha - X_{it}\beta - a_i\theta - b_{\mathrm{J}(i,t)}\psi - k_{i\,\mathrm{J}(i,t)}\mu\right)^2}{2\sigma^2} \right] \right)^{m_{it}} \\ \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \\ \times \prod_{t=1}^{T-1} \left[\delta_{\langle b_{\mathrm{J}(i,t+1)} \rangle | \langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \end{array} \right.$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} \left(\pi_{a\ell}\right)^{a_{i\ell}} \left(\pi_{bm}\right)^{b_{jm}} \left(\pi_{q|\ell m}\right)^{k_{ijq}} \right]$$

Mobility: Separation

Complete Data Likelihood Function

$$L(\rho|Y,Z) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{l} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{\left(\ln w_{i\,\mathrm{J}(i,t)t} - \alpha - X_{it}\beta - a_i\theta - b_{\mathrm{J}(i,t)}\psi - k_{i\,\mathrm{J}(i,t)}\mu\right)^2}{2\sigma^2} \right] \right)^{m_{it}} \\ \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \\ \times \prod_{t=1}^{T-1} \left[\delta_{\langle b_{\mathrm{J}(i,t+1)} \rangle | \langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \end{array} \right.$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} \left(\pi_{a\ell}\right)^{a_{i\ell}} \left(\pi_{bm}\right)^{b_{jm}} \left(\pi_{q|\ell m}\right)^{k_{ijq}} \right]$$

Mobility: Destination

Complete Data Likelihood Function

$$L(\rho|Y,Z) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{c} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[- \frac{\left(\ln w_{i\,\mathrm{J}(i,t)t} - \alpha - X_{it}\beta - a_i\theta - b_{\mathrm{J}(i,t)}\psi - k_{i\,\mathrm{J}(i,t)}\mu \right)^2}{2\sigma^2} \right] \right)^{m_{it}} \\ \times \prod_{t=1}^{T-1} \left[1 - \gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{1-s_{it}} \left[\gamma_{\langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \\ \times \prod_{t=1}^{T-1} \left[\delta_{\langle b_{\mathrm{J}(i,t+1)} \rangle | \langle a_i \rangle \langle b_{\mathrm{J}(i,t)} \rangle \langle k_{i\,\mathrm{J}(i,t)} \rangle} \right]^{s_{it}} \end{array} \right.$$

$$\times \prod_{i=1}^{I} \prod_{j=1}^{J} \left[\prod_{\ell=1}^{L} \prod_{m=1}^{M} \prod_{q=1}^{Q} \left(\pi_{a\ell} \right)^{a_{i\ell}} \left(\pi_{bm} \right)^{b_{jm}} \left(\pi_{q|\ell m} \right)^{k_{ijq}} \right]$$

Earnings

$$L\left(\rho|Y,Z\right) \quad \propto$$

$$\prod_{i=1}^{I} \left\{ \begin{array}{c} \displaystyle\prod_{t=1}^{T}\left(\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left[-\frac{\left(\ln w_{i\ \mathrm{J}(i,t)t}-\alpha-X_{it}\beta-a_i\theta-b_{\mathrm{J}(i,t)}\psi-k_{i\ \mathrm{J}(i,t)}\mu\right)^2}{2\sigma^2}\right]\right)^{m_{it}} \\ \displaystyle\times\prod_{t=1}^{T-1}\left[1-\gamma_{\langle a_i\rangle\left\langle b_{\mathrm{J}(i,t)}\right\rangle\left\langle k_{i\ \mathrm{J}(i,t)}\right\rangle}\right]^{1-s_{it}}\left[\gamma_{\langle a_i\rangle\left\langle b_{\mathrm{J}(i,t)}\right\rangle\left\langle k_{i\ \mathrm{J}(i,t)}\right\rangle}\right]^{s_{it}} \\ \displaystyle\times\prod_{t=1}^{T-1}\left[\delta_{\left\langle b_{\mathrm{J}(i,t+1)}\right\rangle|\langle a_i\rangle\left\langle b_{\mathrm{J}(i,t)}\right\rangle\left\langle k_{i\ \mathrm{J}(i,t)}\right\rangle}\right]^{s_{it}} \end{array} \right.$$

$$\times\prod_{i=1}^{I}\prod_{j=1}^{J}\left[\prod_{\ell=1}^{L}\prod_{m=1}^{M}\prod_{q=1}^{Q}\left(\pi_{a\ell}\right)^{a_{i\ell}}\left(\pi_{bm}\right)^{b_{jm}}\left(\pi_{q|\ell m}\right)^{k_{ijq}}\right]$$

Data: Universe (Frame)

- Matched employer-employee data from the LEHD infrastructure file system (Abowd et al. 2009)
- All individuals employed in IL, IN, WI between 1999-2003 (geographic connectedness)
- Retain the complete history for all such individuals 1990-2010 regardless of location of employer
- 16.9 million persons
- 719 thousand unique employers
- 39 million unique person-employer matches
- Summaries of AKM decomposition (Abowd et al. 2003) provide starting values and benchmarks
- AKM decomposition computed over the universe of all states, except MA, all years 1990-2010, all employers

Data: Estimation Sample

- 0.5% simple random sample of individuals who were employed in IL, IN or WI 1999-2003
- Retain all matches and employers attached to those individuals 1990-2010, and state of employment
- 84,690 Persons
- 181,592 Employers
- 389,718 Matches
- 1,778,490 Person-years (including non-employment spells).

Gibbs Sampler Estimation of Posterior Distributions

Sample wage equation parameters in two steps: (1)

$$\sigma^{(1)} \sim p\left(\sigma|\alpha^{(0)}, \beta^{(0)T}, \theta^{(0)T}, \psi^{(0)T}, \mu^{(0)T}, Z^{(0)}, Y\right)$$

Gibbs Sampler Estimation of Posterior Distributions

Sample wage equation parameters in two steps: (2)

$$\begin{bmatrix} \alpha \\ \beta \\ \theta \\ \psi \\ \mu \end{bmatrix}^{(1)} \sim p\left(\begin{bmatrix} \alpha \\ \beta \\ \theta \\ \psi \\ \mu \end{bmatrix} | Z^{(0)}, \sigma^{(1)}, Y \right)$$

Gibbs Sampler Estimation of Posterior Distributions

Mobility and population parameters sampled independently

$$\gamma^{(1)} \sim p\left(\gamma | Z^{(0)}, Y\right)$$

Gibbs Sampler Estimation of Posterior Distributions

Mobility and population parameters sampled independently

$$\delta^{(1)} \sim p\left(\delta | Z^{(0)}, Y\right)$$

Gibbs Sampler Estimation of Posterior Distributions

Mobility and population parameters sampled independently

$$\pi_a^{(1)} \sim p\left(\pi_a | Z^{(0)}, Y\right)$$

Gibbs Sampler Estimation of Posterior Distributions

Mobility and population parameters sampled independently

$$\pi_b^{(1)} \sim p\left(\pi_b | Z^{(0)}, Y\right)$$

Gibbs Sampler Estimation of Posterior Distributions

Mobility and population parameters sampled independently

$$\pi_{k|ab}^{(1)} \sim p\left(\pi_{k|ab} | Z^{(0)}, Y\right)$$

Gibbs Sampler Estimation of Posterior Distributions

Sample latent data in three steps: (1) Workers

$$[a_1^{(1)}, \ldots, a_I^{(1)}] \sim p\left([a_1, \ldots, a_I] |\, b_0^{(0)}, \ldots, b_J^{(0)},\ k_{11}^{(0)}, \ldots, k_{IJ}^{(0)}, \rho^{(1)}, Y\right)$$

Gibbs Sampler Estimation of Posterior Distributions

Sample latent data in three steps: (2) Firms

$$[b_0^{(1)}, \ldots, b_J^{(1)}] \sim p\left([b_1, \ldots, b_J] |\, k_{11}^{(0)}, \ldots, k_{IJ}^{(0)}, \rho^{(1)}, a_1^{(1)}, \ldots, a_I^{(1)}, Y\right)$$

Gibbs Sampler Estimation of Posterior Distributions

Sample latent data in three steps: (3) Matches

$$[k_{11}^{(1)}, \ldots, k_{IJ}^{(1)}] \sim p\left([k_{11}, \ldots, k_{IJ}] | \rho^{(1)}, a_1^{(1)}, \ldots, a_I^{(1)}, b_0^{(1)}, \ldots, b_J^{(1)}, Y\right)$$

Parallel Computation of the Gibbs Sampler

- Posterior sampling of a_i relies on conditional independence given ρ, b_j and k_{ij}
- Posterior sampling of k_{ij} relies on conditional independence given ρ, b_j and a_i
- Posterior sampling of b_j relies on conditional independence given ρ, a_i and k_{ij} and
  - j and $j\prime$ such that $j\prime \in N(j)$, the set of j neighbors

The last part is the tricky one

Solution for b_j: Graph Coloring

- **Objective:** Label nodes in the *employer projection* of the *RMN* so that no two adjacent nodes have the same label.
- **Analogy:** Map Coloring: Pick smallest number of colors so no two adjacent geographic entities have the same color?
- Determining minimum number of colors is NP-hard
- Application: Partition employer adjacency matrix into structurally orthogonal groups of columns
- Parallel process all employers j in each color
- Finding a small number of colors is "good enough" for this application

Data: Graph $G = (V, E)$ and vertex ordering $\{v_1, \ldots, v_n\}$
Result: Set of colors, Q, and coloring $c : V \to Q$
begin
  Assign v_1 color 1
  for $i \leftarrow 2$ *to* n **do**
    Assign v_i the least-used color not used by any of its neighbors
  end
end

Algorithm 1: Greedy Sequential Coloring

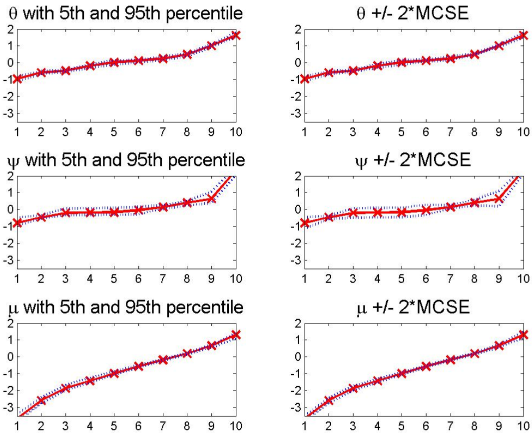
Reference: Gebremedhin et al. (2005)

Properties of Greedy Sequential Coloring

- Worst-case coloring is $\Delta + 1$, where Δ is the maximum degree.
- The actual coloring depends on the vertex ordering sequence input.
- The algorithmic worst-case is bounded above by the maximum number of already-colored nodes connected to the next node in the sequence.
- This bound is minimized by coloring high-degree nodes early.
- Time complexity is $O(m)$ where m is the number of edges.
- The current application used 24 colors.

Exploits network structure together with conditional independence assumptions of the model.

Distribution of Wage Parameters



Latent Class Probabilities: Workers

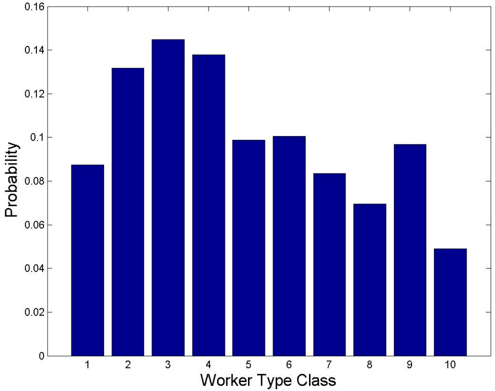


Figure: π_A: Latent Worker Type – Population Probability

Latent Class Probabilities: Employers

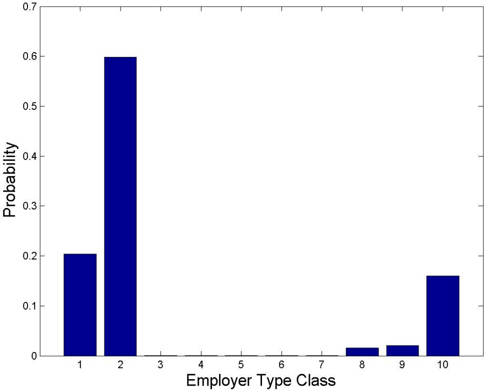


Figure: π_B: Latent Employer Type – Population Probability

Latent Class Probabilities: Matches

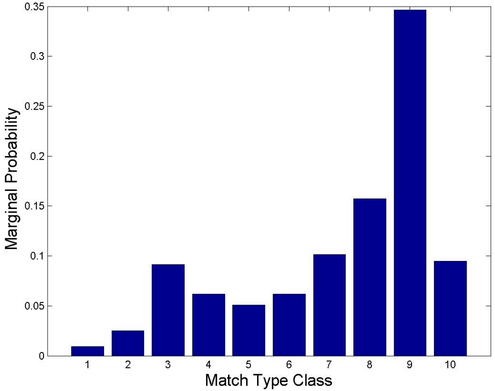


Figure: π_K: Latent Match Type – Marginal Population Probability

Correlation Matrix of Wage Parameters: LEHD Data

	y	$X\beta_{AKM}$	θ_{AKM}	ψ_{AKM}	μ_{AKM}	ε_{AKM}	$X\beta_{Gibbs}$	θ_{Gibbs}	ψ_{Gibbs}	μ_{Gibbs}	ε_{Gibbs}
y	1										
$X\beta_{AKM}$	0.4419	1									
θ_{AKM}	0.3866	-.4865	1								
ψ_{AKM}	0.4981	0.0684	0.1665	1							
μ_{AKM}	0.3362	0.0257	-.0000	-.0028	1						
ε_{AKM}	0.2003	-.0171	-.0000	0.0002	-.0004	1					
$X\beta_{Gibbs}$	0.7767	0.5552	0.2499	0.2443	0.0364	-.0198	1				
θ_{Gibbs}	0.4954	0.1457	0.3841	0.2169	-.0001	0.0021	0.2495	1			
ψ_{Gibbs}	0.2710	0.0151	0.1159	0.4233	0.1129	0.0011	0.0966	0.0430	1		
μ_{Gibbs}	0.0617	0.0462	-.0453	-.0956	0.2766	0.0003	-.0049	-.2271	-.7350	1	
ε_{Gibbs}	0.2687	0.0022	0.0243	0.0758	0.1686	0.7831	0.0001	0.0002	0.0001	0.0000	1

Correlation Matrix of Wage Parameters: LEHD Data

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Steady-state of the Realized Mobility Network

- Let $\lambda_{\ell mq}$ be the measure of type (ℓ, m, q) matches observed in the steady-state
- Define the diagonal matrix

$$\Lambda = diag([\lambda_{111}, \lambda_{112}, \ldots, \lambda_{LMQ}]^T).$$

Note that Λ does not account for transitions to non-employment. In the $2 \times 2 \times 2$ case, Λ is an 8×8 matrix

Define 'type' design matrices analogous to the person, employer, and match design matrices
For the $2 \times 2 \times 2$ model, this matrix is

$$\begin{bmatrix} D & F & G \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}$$

Steady-state of the Realized Mobility Network

Represent theoretical wage offers with an $LMQ \times 1$ vector:
y is the $LMQ \times 1$ vector with

$$y_{\ell mq} = \theta_\ell + \psi_m + \mu_q.$$

Which we rewrite using the type-design matrices:

$$y = D\theta + F\psi + G\mu$$

Steady-state of the Realized Mobility Network

In steady-state, observed log earnings, y, are drawn from a discrete distribution proportional to Λ

$$\Lambda y = \Lambda[DFG][\theta' \psi' \mu']'$$

Network Interpretation of Endogenous Mobility Models

Consider the steady state cross-product matrix:

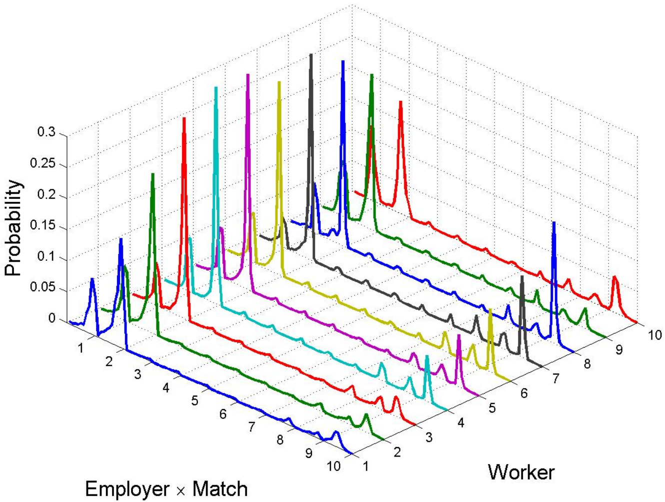
$$\begin{bmatrix} D^T \Lambda D & D^T \Lambda F \\ F^T \Lambda D & F^T \Lambda F \end{bmatrix}$$

This is a model for the adjacency matrix of the realized mobility network.

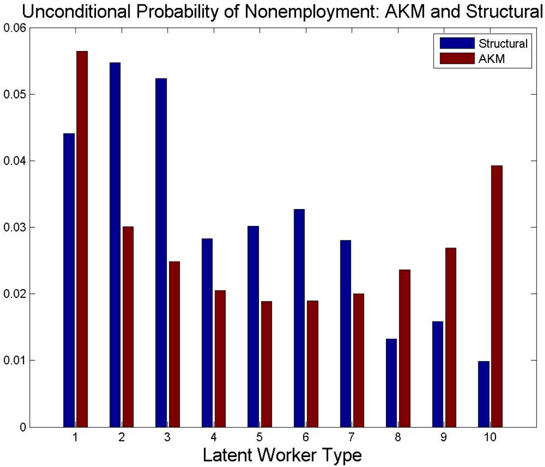
We represent bias in terms of the contrast with the full cross-product matrix

$$\begin{bmatrix} D & F & G \end{bmatrix}^T \Lambda \begin{bmatrix} D & F & G \end{bmatrix} = \begin{bmatrix} D^T \Lambda D & D^T \Lambda F & D^T \Lambda G \\ F^T \Lambda D & F^T \Lambda F & F^T \Lambda G \\ G^T \Lambda D & G^T \Lambda F & G^T \Lambda G \end{bmatrix}$$

Steady-state Firm-match Distribution: Structural



Steady-state Conditional Non-employment Probability



Conclusions

- Endogenous mobility affects the AKM decomposition
- Developed a complete posterior predictive distribution for incorporating endogenous mobility into the AKM wage decomposition
- The Markov transition matrix that describes the evolution of the network adjacency matrix reveals that the probabilities of transitions into better matches do depend on the worker type, firm type and match type in the current job
- Future work will refine the regression-based approach we used here for estimating the expected structural effect given the AKM wage components

Thank You

Variable		Mean	(MCSE)	Variable	Mean	(MCSE)
	age	0.5810	(.0029)	yr1992	−0.0275	(.0008)
	age^2	−0.1880	(.0009)	yr1993	−0.0477	(.0013)
	age^3	0.0277	(.0001)	yr1994	−0.0437	(.0018)
	age^4	−0.0016	(.0000)	yr1995	−0.0352	(.0020)
$female\times$	age	0.0036	(.0007)	yr1996	−0.0225	(.0026)
	age^2	−0.0117	(.0004)	yr1997	0.0036	(.0029)
	age^3	0.0030	(.0001)	yr1998	0.0442	(.0033)
	age^4	−0.0002	(.0000)	yr1999	0.0550	(.0037)
$black\times$	age	−0.0004	(.0012)	yr2000	0.0670	(.0040)
	age^2	−0.0025	(.0007)	yr2001	0.0619	(.0043)
	age^3	0.0005	(.0001)	yr2002	0.0696	(.0046)
	age^4	0.0000	(.0000)	yr2003	0.0659	(.0049)
$hispanic\times$	age	0.0263	(.0008)	yr2004	0.0751	(.0053)
	age^2	−0.0173	(.0009)	yr2005	0.0776	(.0058)
	age^3	0.0029	(.0002)	yr2006	0.0830	(.0062)
	age^4	−0.0001	(.0000)	yr2007	0.0927	(.0065)
	sixq2	0.6879	(.0079)	yr2008	0.0880	(.0069)
	sixq3	1.5227	(.0115)	yr2009	0.0784	(.0074)
	sixq4	2.0854	(.0119)	yr2010	0.0843	(.0077)
	sixq5	2.5327	(.0112)	sigma	0.3659	(.0003)
	sixq6	2.6913	(.0084)			
	sixqleft	−0.0951	(.0031)			
	sixqright	−0.0568	(.0027)			
	sixq4th	0.0989	(.0003)			
	sixqinter	−0.4108	(.0027)			

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